Game Physics

Game and Media Technology
Master Program - Utrecht University

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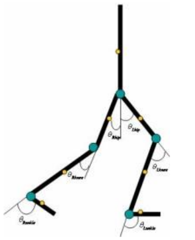
Essential physics for game developers

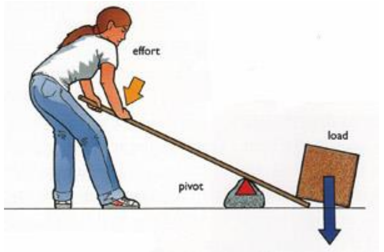
Introduction

The primary issues

- Let’s move virtual objects
  - Kinematics: description of the motion
    - consider position, velocity and acceleration



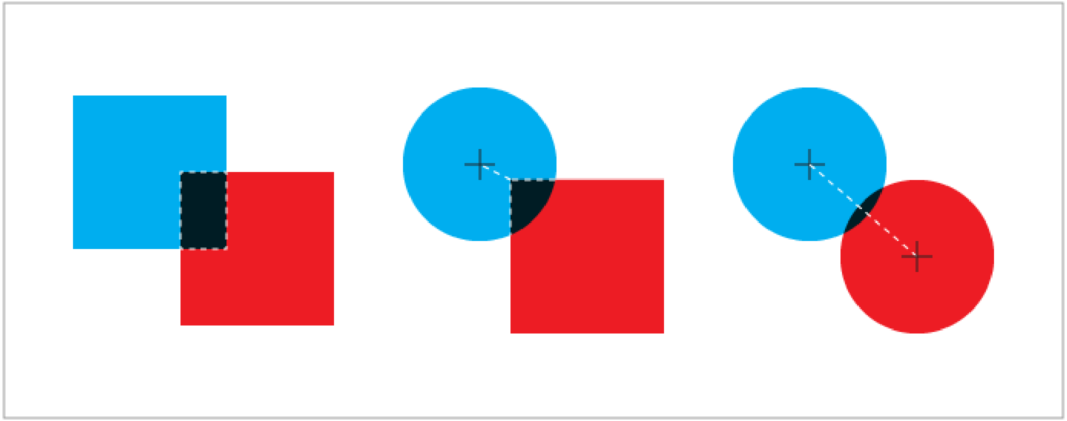
  - Physics: the effect of forces on motion
    - consider mass, inertia and more



- With physics, objects can interact through forces
  - No need to precompute/script motions/interactions
  - React natively according to (implemented) physics laws
  - Good for games!

The primary issues

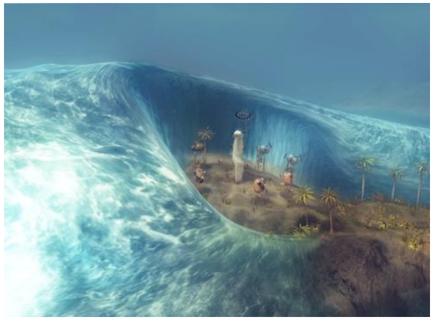
- How do I control the forces to achieve an effect?
- When objects can move, they can also collide
  - Collision detection: did a collision occur and where?
  - Collision resolution: what do we do?



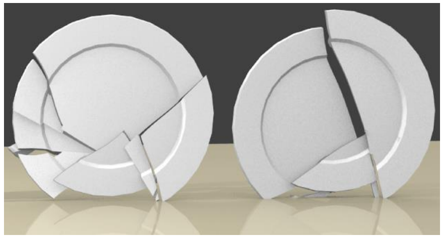
Applications

- We can use physics to simulate
  - rigid bodies
    - mechanically considered as points
    - possibly connected to other bodies

  - soft bodies
    - can deform in a continuum
    - possibly interacting with other bodies



  - breakable bodies
    - acting as single rigid bodies until some events break them into multiple ones



Modeling objects

- In solid body mechanics, an object is represented by a point in space
  - Centroid of the object
    - average of all locations describing the object
  - Center of mass of the object
    - weighted average of density, same as centroid if uniform
- In soft body mechanics, an object is represented by a structured set of points in space
- The geometry/boundary of an object is used in the collision detection and resolution, not in the mechanics

Prerequisites

- We need some basic math knowledge
  - Point
  - Vector
    - magnitude, normalization, addition, subtraction, scalar multiplication, transpose, dot product, cross product ...
    - implicit and parametric representations, orthonormal basis ...
  - Matrix
    - addition, subtraction, multiplication, transpose, dot product, inverse, rank, determinant ...
    - linear and affine transformation, homogeneous coordinate ...
  - Derivative and integral
  - SI units

Kinematics

Kinematics

- To determine the object position p_o at time t
- Suppose we have a constant velocity v
  - Then $p_o(t+\Delta t) = p_o(t) + v\Delta t$
  - Alternatively, $\Delta p_o = p_o(t+\Delta t) - p_o(t) = v\Delta t$
  - If $p_o(0) = P$, then $p_o(t) = P + vt$
  - So we could calculate the object position at any time
- But v is probably not constant over time
  - The velocity is a function of the time $v(t)$ responding to external forces applied, so $p_o(s) = P + \int_0^s v(t)\,dt$
  - The position has to be calculated regularly according to the (change in) velocity

Kinematics

- Similar properties hold for acceleration a
  - If constant acceleration, then $\Delta v = a\Delta t$
  - If not (probably), then $v(s) = \mathrm{V} + \int_0^s a(t)\, dt$
- But do we really need to integrate?
  - If we recalculate the position at each game loop called after Δt
  - And we assume velocity and acceleration were constant in that time interval (typically very short, around few milliseconds or less)
  - Then we can use the formulas $p_o(t + \Delta t) = p_o(t) + v\Delta t$ and $v(t + \Delta t) = v(t) + a\Delta t$

Equations of motion

1.1-1.5

- From this we can derive the following equations

$$v(t + \Delta t) = v(t) + a\Delta t$$

$$\bar{v} = \frac{v(t + \Delta t) + v(t)}{2}$$

$$\Delta p_o = {}^{1}\!/_{2}\,(v(t + \Delta t) + v(t))\Delta t$$

$$\Delta p_o = v(t)\Delta t + {}^{1}\!/_{2}\, a\, \Delta t^2$$

$$v(t + \Delta t)^2 = v(t)^2 + 2a\Delta p_o$$

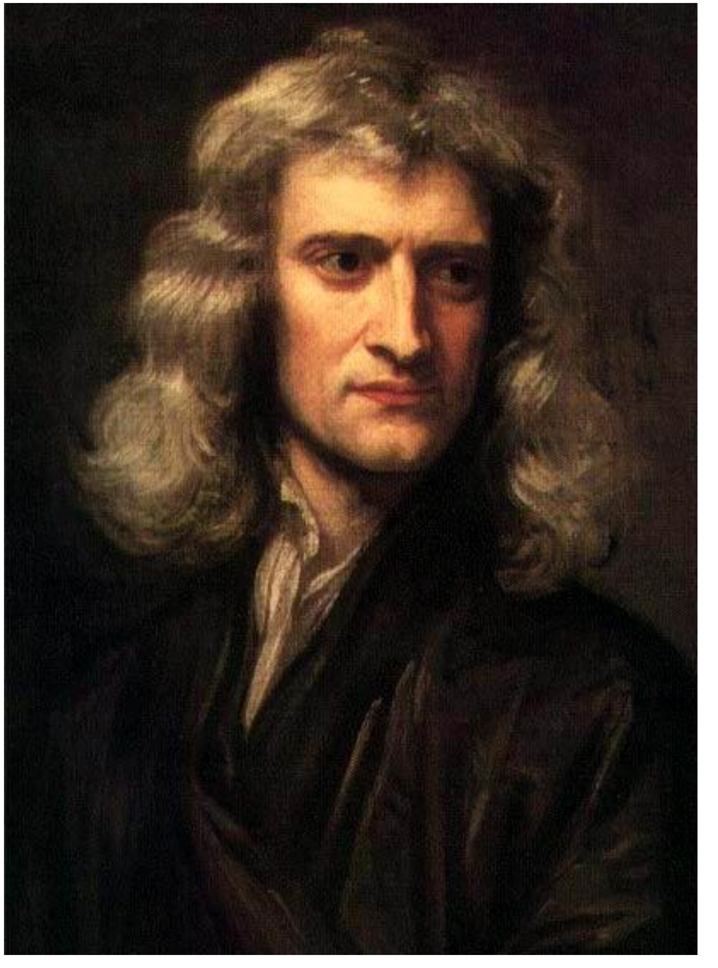
Physics-based system

Forces

- In a physics game engine what cause motion are not (change in) velocities or accelerations but forces
- The sum of all forces acting on an object determines how it moves
- The notation F will be used in place of $\vec{F}$, but the context indicates if it is a scalar or a vector

Newton's laws of motion

- In the late 17^{th} century, Sir Isaac Newton described three laws that govern all motion on Earth



First Law of Motion

- Newton's first law specifies what happens when the net force on an object is null: all the individual forces cancel each other out, there should be no change in the motion

$$\boxed{\textit{If } F_{net} = 0, \textit{ there is no change in motion}}$$

- It means that, until a non-zero net force is applied
  - an object at rest stays at rest
  - an moving object continues to move at the same speed in the same direction

Second Law of Motion

- Newton's second law of motion describes how an object moves when the net force is not zero

$$F_{net} = m * a$$

where m is the mass and a the acceleration

- It also shows that
  - the more force is applied, the faster the object speeds up
  - for an equal amount of force, lighter objects speed up faster than heavier objects

Third Law of Motion

- Newton's third law of motion describes the relationship between forces

For every force there is an equal and opposite force, or, when two objects come into contact they exert equal and opposite forces upon each other

Forces

Gravity

- Newton's Law of Gravitation states that the gravitation force between two masses A and B is
  - proportional to the product of the masses m_A and m_B
  - inversely proportional to the square of the distance between the two masses
  - acting on the line connecting the two masses

$$F_g = F_{A\to B} = -F_{B\to A} = G\frac{m_A m_B}{r^2}u_{AB}$$

where G is the gravitational constant $6.673 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$, r is the distance between the two masses and u_{AB} the unit distance vector

Gravity on Earth

- By applying Newton's second law to an object with mass m on the surface of the Earth we obtain

$$F_{net} = F_g = m * a$$

$$G \frac{m * m_{Earth}}{{r_{Earth}}^2} = m * a$$

$$G \frac{m_{Earth}}{{r_{Earth}}^2} = a$$

$$a = 6.673 \times 10^{-11} \frac{5.98 \times 10^{24}}{(6.377 \times 10^6)^2} = 9.81\, m/s^2$$

Other gravity

- On Earth at altitude h: $a = G\frac{m_{Earth}}{(r_{Earth}+h)^2}$
- On the Moon
  - $m_{moon} = 7.35 \times 10^{22}\ kg$
  - $r_{moon} = 1738\ km$
  - $g_{moon} = 1.62\ m/s^2$
- On Mars
  - $m_{mars} = 6.42 \times 10^{23}\ kg$
  - $r_{mars} = 3403\ km$
  - $g_{mars} = 3.69\ m/s^2$

Weight

1.6-1.9

- Other name for the gravitation force

$$W = m * g$$

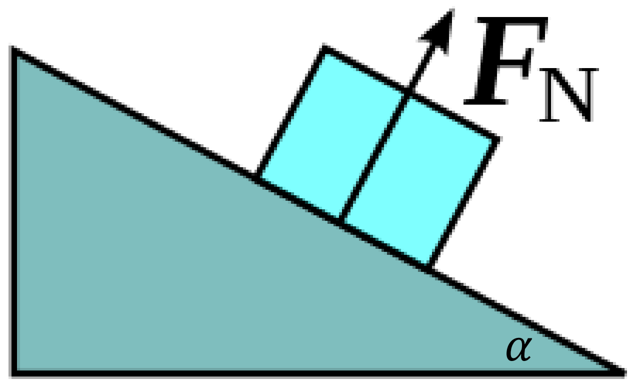
where m is the mass of the object and g the acceleration due to gravity

- We note that force unit is $kg * m/s^2$, noted as Newtons (N)

Normal force

1.10

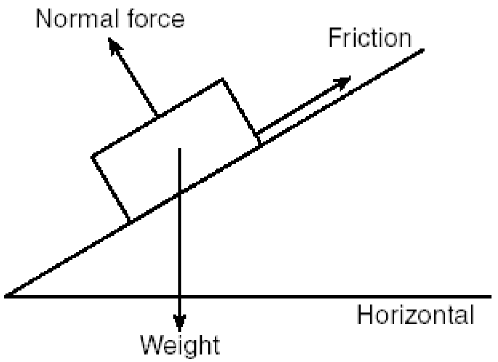
- Force acting as a reaction to a contact
  - Direction normal to the surface of contact
  - Magnitude equals to the incoming force projected on the normal direction



  - Here, $F_N = W\cos(\alpha)$
- Related to collision handling (more later)

Friction

- What about the tangential component of the reaction to contact?
  - Here, the $W\sin(\alpha)$
  - Energy is conserved, so where does it go?
- In the friction force, *i.e.* ability to resist movement
  - Static friction keeps an object on a surface from moving
  - Kinetic friction slows down an object in contact



Friction

- If added up forces are less than the static friction then the object will not move
- As soon as the object starts moving (because forces are higher than static friction), then the kinetic friction takes over
- Both friction forces depend on the two surfaces coming in contact
  - the ‘smoother’ the surfaces, the less friction
  - defined by a coefficient of friction μ, the ratio of the force of friction between two bodies and the force pressing them together

Friction

Surface Friction	**Static (μ_s)**	**Kinetic (μ_k)**
Steel on steel (dry)	0.6	0.4
Steel on steel (greasy)	0.1	0.05
Teflon on steel	0.041	0.04
Brake lining on cast iron	0.4	0.3
Rubber on concrete (dry)	1.0	0.9
Rubber on concrete (wet)	0.30	0.25
Metal on ice	0.022	0.02
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Nickel on nickel	1.1	0.53
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53

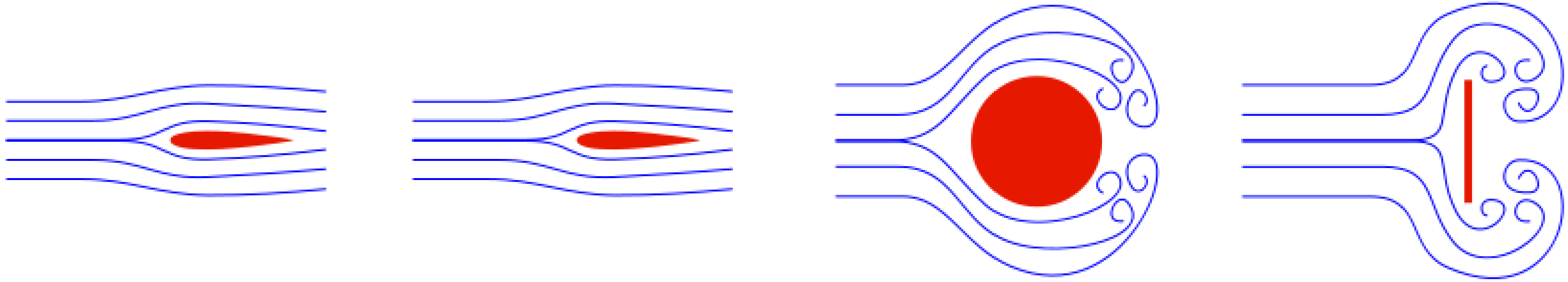
Friction

1.11-1.12

- Friction forces are calculated from these coefficients
  - Static friction: $F_s = \mu_s F_N$
  - Kinetic friction: $F_k = \mu_k F_N$
- The kinetic coefficient of friction is always smaller than the static coefficient of friction
- If the tangential force is larger than the static friction, the object moves
- If the object moves while in contact, the kinetic friction is applied to the object

Fluid resistance

- An object moving in a fluid (air is a fluid) is slowed down by this fluid
- This is called fluid resistance or drag and depends on several parameters
  - *e.g.* the faster the object, the larger resistance
  - *e.g.* the more exposed surface, the larger resistance



Fluid resistance

- At high velocity, the drag force $F_{D_{high}}$ is quadratic to the relative velocity of the object v

$$F_{D_{high}} = -\frac{1}{2} * \rho * v^2 * C_d * A$$

where ρ is the density of the fluid (1.204 for air at $20℃$), C_d is the drag coefficient (depends on the shape of the object) and A is the reference area (area of the projection of the exposed shape)

Fluid resistance

1.13

- At low velocity, the drag force is considered linearly proportional to the velocity

$$F_{D_{low}} = -b * v$$

where b depends on the properties of the fluid and the shape of the object

- High/low velocity threshold is defined by Reynolds Number (Re)
- Approximation of real physics, just good for games

Buoyancy

1.14

- The buoyancy is the force that develops when an object is immersed in a fluid
- It is a function of the volume of the object V and the density of the fluid ρ

$$F_B = \rho * g * V$$

- It comes from the difference of pressure above and below the immerged object and is directed straight up, counteracting the weight of the object

Springs

- Springs are used to connect two or more objects
- They react according to Hook's Law on extension and compression, *i.e.* on the relative displacement between the connected objects
- The relative length l to the rest-length l_0 determines the force to apply

$$F_s = -K(l - l_0)$$

where K is the spring constant (in N/m)

Dampers

- Dampers try to slow down the motion between two objects A and B connected by a spring (reduce amplitude of oscillation)
- The damping force depends on the relative speed between the two objects

$$F_C = -C(v_A - v_B)$$

where C is the damping coefficient and the resulting force applied on A (opposite on B)
- Similar to drag force at low velocity!

Free Body Diagram

- Once you have all the forces acting on an object, you have to sum them up and divide them by the mass of the object to obtain the acceleration

$$F_{net} = m * a$$

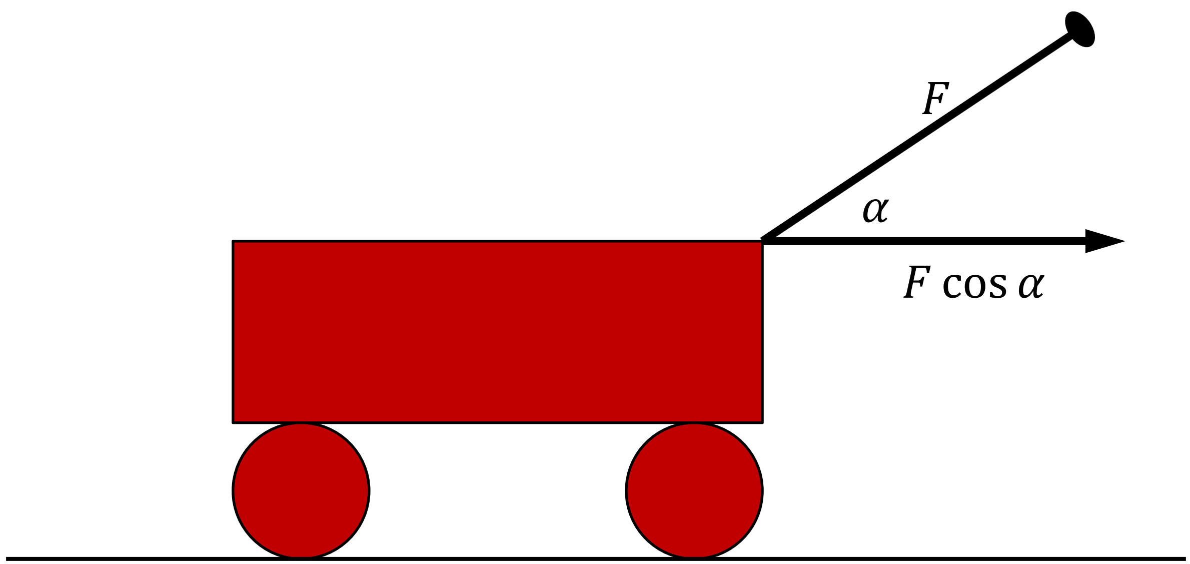
- To help you to analyze the forces, you can use a pictorial device, called Free Body Diagram that includes
  - the shape of the object, its center of mass and contact points
  - the external forces with direction, magnitude and point of application

Energy-based system

Work

- The work W (in $Joule = N * m$) is defined as the amount of displacement Δp_o of an object times the net force F applied on the object in the direction of the displacement

$$W = F * \Delta p_o$$



Kinetic energy

- The kinetic energy E_K is the amount of energy an object has from moving

$$E_K = \frac{1}{2}\, m\, v^2$$

where m is the mass of the object and v its speed

  - the faster the object is moving, the more energy it has
  - the energy is a scalar (that's why speed is used and not velocity)
  - unit is also Joule as $kg(m/s)^2 = \left(kg * \frac{m}{s^2}\right) m = N * m = J$

Work-Energy theorem

1.15

- The Work-Energy theorem states that the net work of an object is equal to the change in its kinetic energy

$$W = \Delta E_K = E_K(t + \Delta t) - E_K(t)$$

i.e.

$$F\,\Delta p_o = \frac{1}{2} m\left(v(t + \Delta t)^2 - v(t)^2\right)$$

  - We can notice that if a work exists, the object either speeds up or slows down (difference of velocities not null)
  - Very similar to Newton's second law...

Potential energy

- (Gravitational) Potential energy is the energy 'stored' in an object due to its height off the ground

$$E_P = m * g * h$$

  - Simple product of the weight $W = m * g$ and height h
  - Also measured in Joules (as here $kg * \frac{m}{s^2} * m$)

- Other potential energies exist but are insignificant in game physics

Conservation of mechanical energy

- The law of conservation of mechanical energy states that the energy of an object cannot be created or destroyed
  - but can switch forms, including the two we just described

$$E_K(t + \Delta t) + E_P(t + \Delta t) = E_K(t) + E_P(t)$$

i.e.

$$\frac{1}{2}mv(t + \Delta t)^2 + mgh(t + \Delta t) = \frac{1}{2}mv(t)^2 + mgh(t)$$

Conservation of mechanical energy

- Picture a roller coaster cart at the top of the first hill
  - It has a lot of potential energy but only a little bit of kinetic energy
  - Then it goes down the first big drop, and as it loses height, the cart picks up speed
  - By the time it gets down to the bottom of the hill, almost all the potential energy has switched to kinetic, and now the cart is at its maximum speed

Conservation of mechanical energy

1.16

- In reality, other external forces can be applied such as friction and air resistance
  - Reduce the total energy to conserve over time
  - Actually converted into heat and air displacements (sound waves), so law is correct (equation not complete)
- We can compensate by adding an extra term E_O to the equation

$$E_K(t+\Delta t)+E_P(t+\Delta t)=E_K(t)+E_P(t)+E_O$$

  - if $E_O > 0$, some energy has been 'lost'
  - In game engines E_O is usually quite small and depends at least on velocity (to avoid creating energy)

Momentum

- The linear momentum p is defined as the mass of an object multiplied by its velocity
$$p = m * v$$
  - the heavier the object, the more momentum, *i.e.* the more difficult it is to stop
  - same trend for higher velocity
  - as velocity changes constantly, we talk about instantaneous momentum or change over a time interval
  - unit is $kg * m/s$ (different from a force)
  - vector in the direction of the velocity

Impulse

- From Newton's second law $F = m * a$

$$\Leftrightarrow F = \frac{m\big(v(t+\Delta t) - v(t)\big)}{\Delta t}$$

$$\Leftrightarrow F = \frac{m * v(t+\Delta t) - m * v(t)}{\Delta t}$$

$$\Leftrightarrow F = \frac{\Delta p}{\Delta t} \Leftrightarrow F\Delta t = \Delta p$$

- An impulse is the rate of change of the momentum, *i.e.* the net force applied on the object multiplied by the amount of time of application
  - an impulse is a force delivered in a very small amount of time, considered as an instant
  - unit is also $kg * m/s$ (so also different from a force)

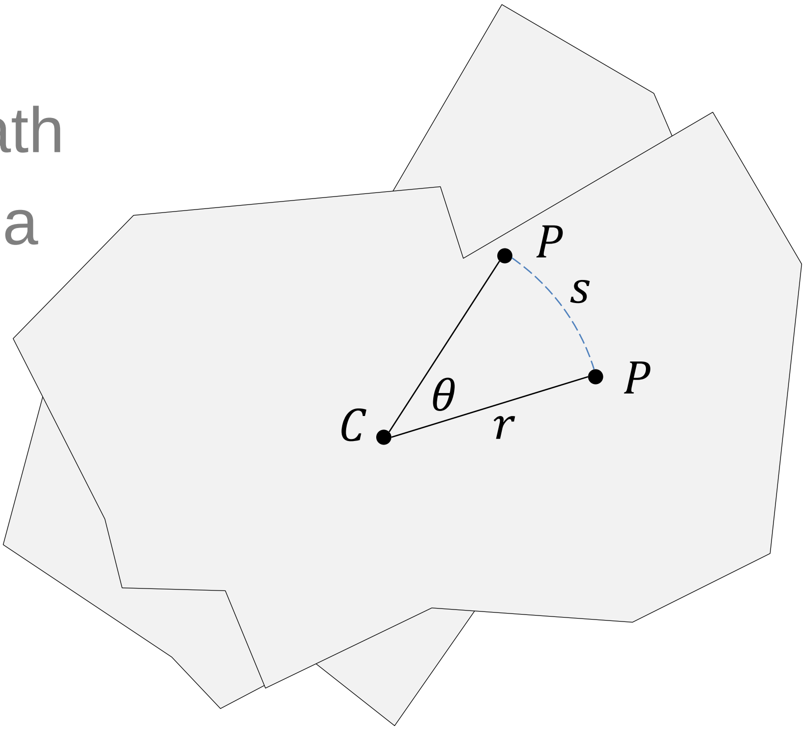
Rotational motion

Rotational motion

- So far our point object moved along a trajectory and forces were applied on that point object
- But you want to move actual dimensional objects and forces applied on them make them move and rotate

Circular motion

- Imagine the following object where C is the center of rotation, P a point belonging to the object and r the distance between C and P
  - when the object rotates P travels along a circular path
  - after Δt, P has moved by a distance s



Angular displacement

- This angle θ represents the rotation of the object

$$\theta = s/r$$

where s is the arc length and r the radius

  - unit is radian (rad)
  - note that 1 radian is the angle describing an arc length of 1 at a distance 1

Angular velocity

- The angular velocity is the rate of change of the angular displacement

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta(t + \Delta t) - \theta(t)}{\Delta t}$$

  - unit is rad/s

Angular acceleration

- As linear acceleration was the rate of change of the linear velocity, the angular acceleration is the rate of change of the angular velocity

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega(t + \Delta t) - \omega(t)}{\Delta t}$$

  – unit is rad/s^2

Equations of motion

- The five equations stand for rotation as well:

$$\omega(t + \Delta t) = \omega(t) + \alpha \Delta t$$

$$\bar{\omega} = \frac{\omega(t + \Delta t) + \omega(t)}{2}$$

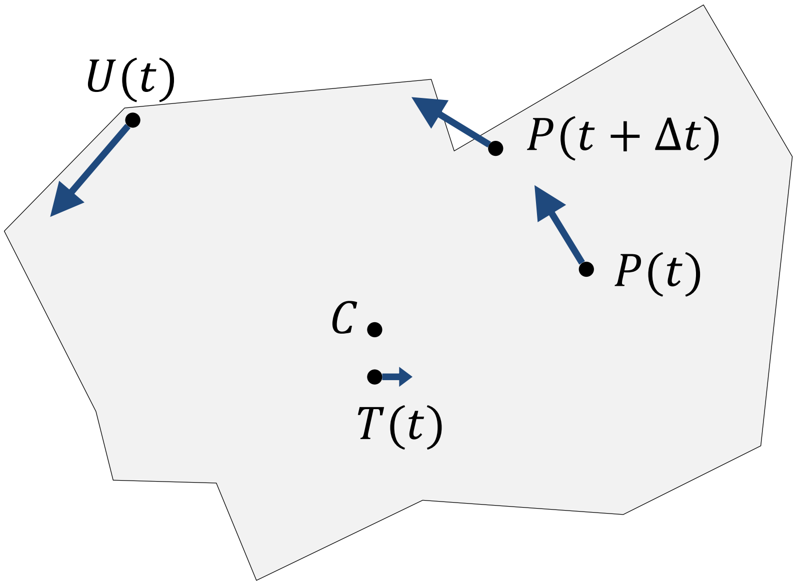
$$\Delta\theta = {}^{1}\!/_{2}\,(\omega(t + \Delta t) + \omega(t))\Delta t$$

$$\Delta\theta = \omega(t)\Delta t + {}^{1}\!/_{2}\,\alpha\,\Delta t^2$$

$$\omega(t + \Delta t)^2 = \omega(t)^2 + 2\alpha\Delta\theta$$

Linear and angular velocities

- Every point on a rigid body moves with the same angular velocity
- Different points on a rigid body can have different linear velocities



Tangential velocity

- As $\theta = s/r$ and r does not vary over time, we have $\Delta\theta = \Delta s/r$
- We also have by definition $\omega = \Delta\theta/\Delta t$
- So $\omega = \frac{1}{r} * \frac{\Delta s}{\Delta t}$
- But for small Δt, Δs can be seen as a line and not anymore as an arc, so we can use the linear velocity at P to estimate $\frac{\Delta s}{\Delta t}$
- This velocity is called tangential velocity (because tangent to the circular path) and noted v_t
- Therefore, for small Δt, we have $\omega = \frac{v_t}{r}$

Tangential acceleration

1.18-1.19

- We can of course defined the tangential acceleration the same way

$$a_t = \alpha * r$$

where α is the angular acceleration

  - Similarly to the velocity equation $v_t = \omega * r$

Centripetal acceleration

- Besides the tangential acceleration, there is also the centripetal acceleration of a point on the object

$$a_n = \frac{{v_t}^2}{r} = r\omega^2$$

- This acceleration is directed towards the axis of rotation
  - opposite to centrifugal acceleration

Rotational dynamics

- So what causes rotational motion?

➢ Forces not applied on the center of mass but on any point belonging to the rigid body
  - Produce a rotation about the center of mass (COM)

Torque

- Suppose a tangential force is applied on a point P belonging to an object, then $F_t = m * a_t$
- If we multiply by the distance r to COM

$$F_t * r = m * a_t * r$$

- But we know that $a_t = r * \alpha$
- So we have $F_t * r = m * (r * \alpha) * r = m * r^2 * \alpha$

Torque

1.20

- This defines the torque of F_t applied to a point at a distance r from the COM

$$\tau = m * r^2 * \alpha$$

where m is the mass, r the distance to COM and α the angular acceleration of the object

  – unit is $N * m$

- The torque rotates an object about its axis of rotation through the center of mass

Torque

- But a force is in general not applied in the direction of the tangent
- The torque τ is then defined as

$$\tau = r \times F$$

  where r is the vector from the COM to the point where F is applied
- $\times$ is the cross product, so the direction of the torque is perpendicular to both F and r

Newton's Second Law

- The law $F = m * a$ has an equivalent with rotation and torque

$$\tau = I * \alpha$$

- I is called the inertia
  - more on its form later

- Torque causes angular acceleration where force causes linear acceleration

Rotational kinetic energy

- We can also translate the energy formulas to rotational motion
- We have the rotational kinetic energy defined as

$$E_{Kr} = \frac{1}{2} * I * \omega^2$$

where I is the inertia and ω the angular velocity

Conservation of mechanical energy

- The principle of mechanical energy conservation holds, but one more type of energy must be considered

$$
\begin{aligned}
E_{Kt}(t+\Delta t) + E_P(t+\Delta t) + E_{Kr}(t+\Delta t) \\
= E_{Kt}(t) + E_P(t) + E_{Kr}(t) + E_O
\end{aligned}
$$

where E_{Kt} is the translational kinetic energy, E_P is the potential energy, E_{Kr} is the rotational kinetic energy and E_O the other energies (surface friction, air resistance etc.)

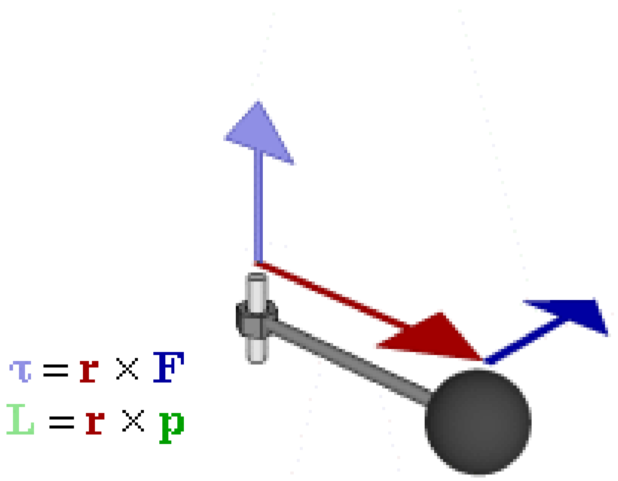
Angular momentum

- Rotational motion also produces angular momentum about the COM

$$L = r \times p$$

where r is the distance vector and p is the linear momentum

  – unit is $N * m * s$



Angular momentum

- For an object with a fixed mass that is rotating about a fixed symmetry axis, the angular momentum is expressed as the product of the moment of inertia of the object and its angular velocity

$$L = I * \omega$$

  - Again, the more inertia the object has, the more difficult it is to stop the object to rotate
  - And the more velocity it has, the more difficult it is as well

Impulse

- We can also apply off-center forces for a very short amount of time
- Such ‘angular’ impulse results in a change in angular momentum, *i.e.* in angular velocity

$$\tau \Delta t = \Delta L$$

End of
Essential physics
for game developer

Next

Rigid body physics